Energy Efficient Stochastic Computing with Sobol Sequences

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Abstract

❑ In this paper, the Sobol sequence is considered for energy-efficient stochastic computing (SC) circuits.

❑ The use of Sobol sequences improves the output accuracy of a stochastic circuit with a reduced sequence length compared to the use of the Halton sequences and pseudorandom sequences due to a lower discrepancy.

❑ Sobol sequence-based SC elements cost less energy than the Halton-based counterparts when multiple random sequences are required in a circuit.

Introduction

Stochastic Computing (SC)

In SC, information is carried by a stochastic bit stream.
For example: Bit stream: {0,1,0,1,1,0,0}, coding 3/7.

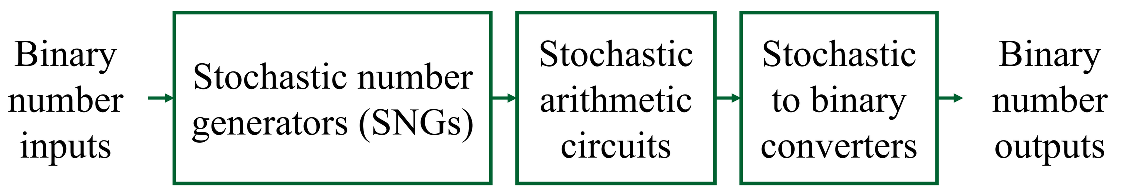


Fig. 1. A typical stochastic computing system.

Stochastic number generators (SNGs)

The component that converts a real number to a stochastic bit stream is usually referred to as an SNG.

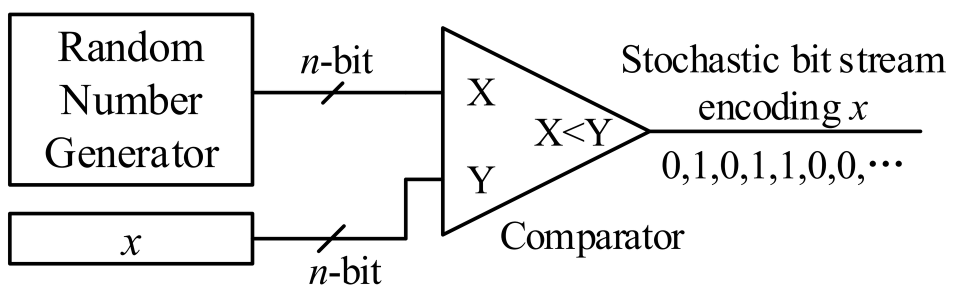


Fig. 2. A stochastic number generator (SNG).

Random number generators (RNGs)

Conventionally, an RNG can be implemented by a linear feedback shift register (LFSR).

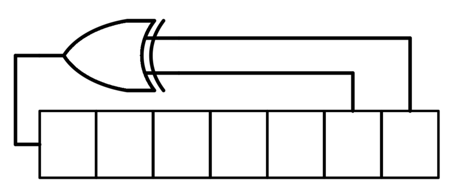
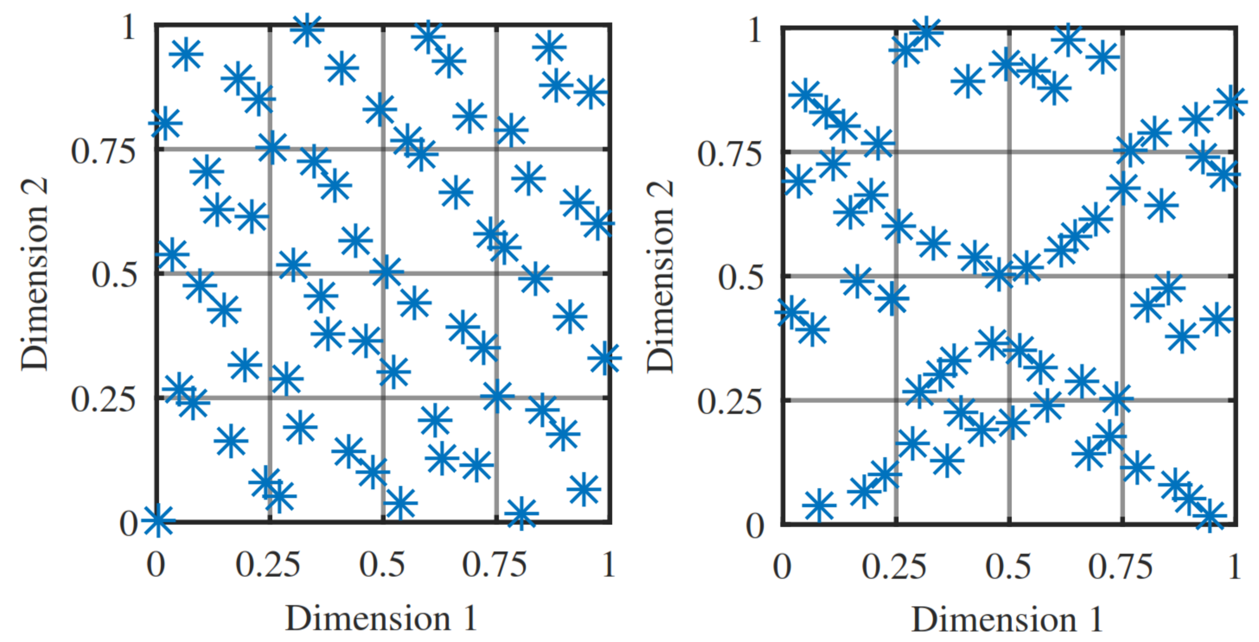


Fig. 3. An LFSR.

Limitations: low accuracy, long latency [1]

Low-discrepancy sequences



(a) Sobol sequences (b) Pseudorandom sequences

Fig. 4. An example of two-dimensional (a) Sobol sequences and (b) Pseudorandom sequences generated by LFSRs. Sobol sequences are more evenly distributed than pseudorandom sequences.

❑ Discrepancy describes how evenly a random sequence is distributed in the sample space. It has been shown that a lower discrepancy indicates a smaller error in a Monte-Carlo (MC) integration [2].

❑ Since SC can be considered as an MC problem, a lower discrepancy also implies a smaller error in an SC circuit [3].

Table 1. Comparison of LD sequences and LFSR-generated sequences

	Low-discrepancy (LD) sequences	LFSR-generated sequences
Known as	Quasi-random sequences	Pseudorandom sequences
Distribution	Evenly	Irregularly
Error	$O((\log L)^{s-1}/L)$*	$O(1/\sqrt{L})$

* L is the sequence length and s is the dimension of the sample space, i.e., the number of independent stochastic sequences in SC.

❑ Sobol sequences are preferred since they are base-2 (binary) LD sequences, with no base conversion required.

Sobol sequence generator

Algorithm: Sobol sequence generation [4]

1: $R_0 = 0$
2: for $i = 0$ to $L-2$ do
3: k= least significant zero position of i
4: $R_{i+1} = R_i \oplus V_k$ (the kth direction vector)
5: end for
Return $\{R_i\}$, $i = 0, 1, \ldots, L-1$

Table 2. Example of a direction vector array

k	V_k
0	10000000
1	11000000
2	11100000
3	11110000
…	

Example:

$R_0 = (0.00000000)_2$

Next:

$i = 0 \to k = 0 \to V_0 = 10000000$

$R_1 = R_0 \oplus V_0 = (0.10000000)_2$

Next:

$i = 1 \to k = 1 \to V_1 = 11000000$

$R_2 = R_1 \oplus V_2 = (0.01000000)_2$

Next:

$i = 2 \to k = 0 \to V_0 = 10000000$

$R_3 = R_2 \oplus V_0 = (0.11000000)_2$

…

❑ A design for the Sobol sequence-based SNG is shown in Fig. 5 [5].

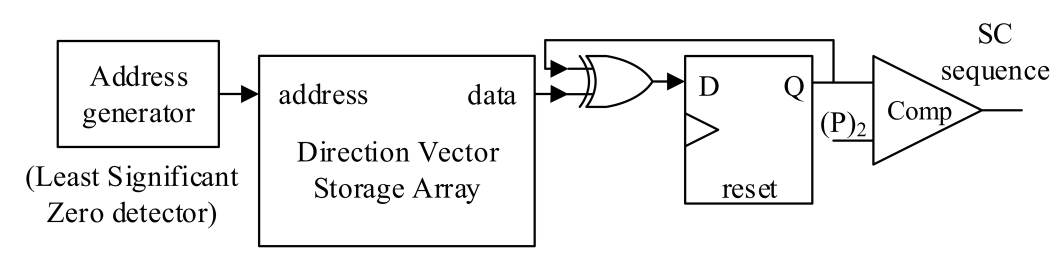


Fig. 5. A Sobol sequence-based SNG.

Hardware Simulation and Analysis

Basic SC elements

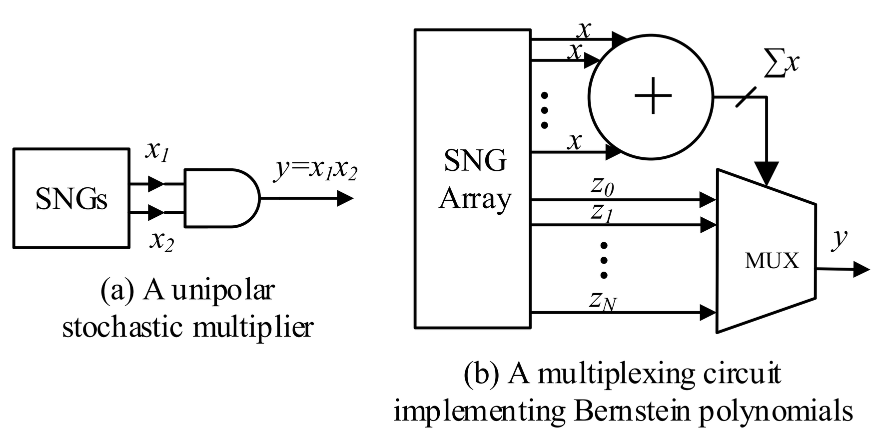


Fig. 6. Basic stochastic computing elements.

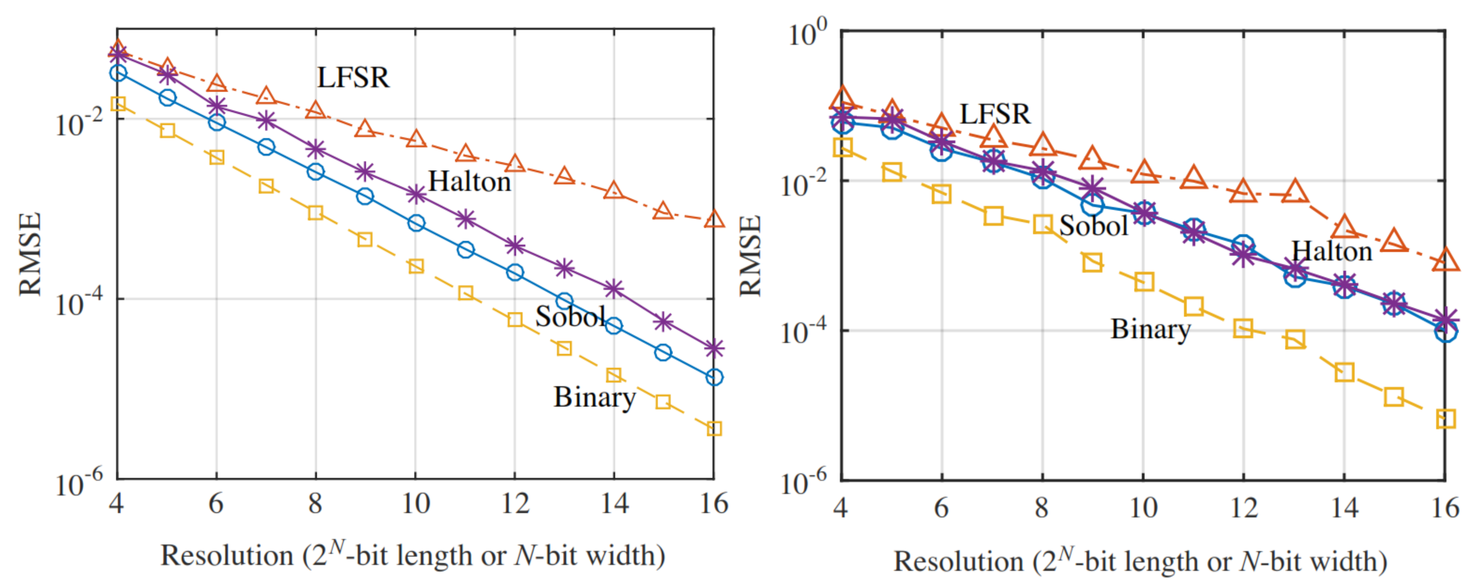
Bernstein polynomial [6]:

$$y = B(x) = \sum_{i=0}^{n} z_i B_{i,n}(x)$$

$$B_{i,n}(x) = \binom{n}{i} x^i (1-x)^{n-i}$$

A 3rd order polynomial stochastic circuit is implemented: $y = ax^3 + bx^2 + cx + d$.

Accuracy comparison



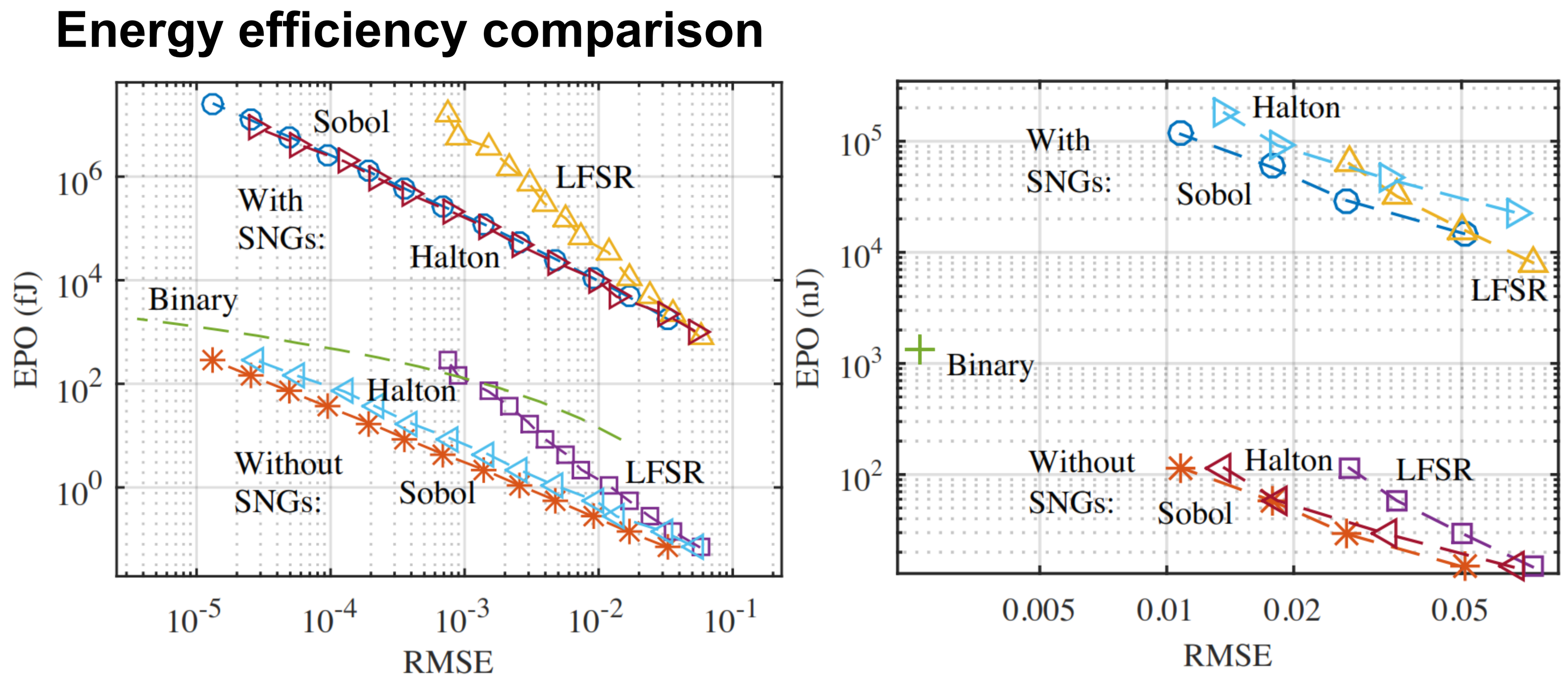
(a) Multiplier (b) Bernstein polynomial

Fig. 7. Accuracy comparison using sequences with different lengths.

❑ Sobol sequence-based designs generally have the best accuracy using the same sequence length, while LFSRs result in the least accurate outputs.

❑ For the unipolar stochastic multiplier, Sobol sequences can achieve a similar accuracy as Halton sequences with half of the sequence length.

Energy efficiency comparison



(a) Multiplier (b) Bernstein polynomial

Fig. 8. Energy per operation (EPO) comparison using different sequences.

❑ Sobol sequence-based designs can achieve better energy efficiency in most cases, especially for the Bernstein polynomial circuit (multiple independent stochastic sequences are required).

Energy per Operation (EPO) for SC:

EPO = Sequence length × power × clock period

Conclusion

❑ The use of Sobol sequences can lead to a similar or higher accuracy than using pseudorandom sequences and Halton sequences for the same sequence length.

❑ When multiple independent random sequences are required, the use of Sobol sequences can reduce the energy consumption compared to the other two random sequences.

Reference

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[4] Bratley, Paul, and Bennett L. Fox. "Algorithm 659: Implementing Sobol's quasirandom sequence generator." ACM Transactions on Mathematical Software 14.1 (1988): 88-100.

[5] Dalal, Ishaan L., Deian Stefan, and Jared Harwayne-Gidansky. "Low discrepancy sequences for Monte Carlo simulations on reconfigurable platforms." ASAP, 2008.

[6] Qian, Weikang, et al. "An architecture for fault-tolerant computation with stochastic logic." IEEE Transactions on Computers 60.1 (2011): 93-105.